FLARE STARS[1]

Introduction

Historically the first flares of brightness in some red dwarf flare stars were observed as early as in the first quarter of our century [1]. However, those stars attracted attention much later, after the discovery and study by W. Luyten [2] and A. Joy and M. Humason [3] of the flares of a prototype of this class of variable stars, i.e., UV Ceti at the close of the 1940s.

A considerable number of flare stars has been discovered over a comparatively short period in the circumsolar volume and in stellar aggregates — associations and clusters. At present the number of discovered flare stars is well over 600. In the Pleiades cluster and the Orion association alone over 500 flare stars have been found [4-6] and about 40 in the vicinity of the Sun [7]. As a result of the growing interest in flare stars, international campaigns were sponsored in recent years to make regular photographic observations of flare stars in some stellar associations and clusters and photoelectric observations of particular flare stars around the Sun in order to study them at great length.

The campaign, the active participants of which are the observatories of Armagh, Asiago, Boyden, Byurakan, Cerro Tololo, the Crimea, Catania, Tokyo, Tonanzintla and others, has substantially amplified our notions of flare stars.

It turned out that the flare phenomenon is more akin to explosions than could be expected from the first observations. Photoelectric observations with high resolving power in time have shown [8] that sometimes it takes

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[1] Jointly with L. V. Mirzoyan.

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the star only several seconds to go through a period of increasing brightness while a decrease in brightness may continue over a long period.

In the present paper an attempt is made to consider several sets of data on flare stars from the standpoint of those ideas on the evolution of stars that are advanced at Byurakan (see, for instance, [9]).

The existence of close relations between flare stars and stars of the T Tauri type is an established fact nowadays. More proof in support of such relations has been produced by one of the authors of the present paper [10, 11] as well as by G. Haro [12, 13].

It was proved at Byurakan as early as in 1953 [10, 11] that the unusual features of radiation of T Tauri and UV Ceti-type stars, and the frequent appearance in their spectra of excessive shortwave radiation are signs typical of the earliest stages in the development of low luminosity stars.

Soon the discovery by Haro and his associates [12] of flare stars in stellar aggregates (associations and clusters) corroborated this point of view spanning a bridge between the stars of the T Tauri and UV Ceti types. Then the most typical flares were observed in certain stars of the T Tauri type. It became evident, in the light of those discoveries, that all the above stars form a wide class of comparatively young nonstable objects.

G. Haro [12, 13] was the first to appreciate the tremendous significance of flare stars in the picture of the earliest stages of evolution of dwarf stars. Relying on observational data concerning flare stars in stellar clusters and associations, he came to the major conclusion that the earlier stage in the evolution of dwarf stars, i.e., the stage of T Tauri (or RW Aurigae) is followed, roughly speaking, by a stage characterized by the ability to produce flares of considerable power from time to time.

An appreciable contribution to the discovery and study of flare stars has been rendered by L. Rosino and his associates [14, 15] who confirmed many of Haro's results. This line was further developed in investigations conducted at the Byurakan Observatory [4, 16, 17].

The abundance of flare stars in stellar aggregates

The distribution of flare stars in the Galaxy, at least in the region available for the observation of stars of low luminosity, is quite inhomogeneous. The investigations of Haro, Rosino and their associates [12 – 15, 18, 19] have established the fact that like stars of the RW Aurigae type, flare stars tend to form groups located in stellar associations and in comparatively young clusters. In the general stellar field, at least among the dwarfs of G, K and earlier M types, flare stars are quite few in number. There are some indications, although of no decisive value, to the effect that classical flare stars of the UV Ceti type around the Sun likewise form a physical system [1, 12, 20, 21]. Some definite evidence favoring this view has been obtained in a recent paper of M. A. Arakelian [21].

When the number of flare stars discovered in some aggregates (Pleiades, Orion) reached several dozen, the question of their total number in those systems came to the fore. A solution of this problem seemed feasible by comparing the number of those stars of the system, for which one single flare was observed, with the number of stars for which recurrent flares were observed. Of course, in this case certain assumptions are made concerning the distribution of flares of a given star in time (for instance, on Poisson distribution of the flare moments).

The statistical estimate of the total number of flare stars in the Pleiades cluster being unexpectedly great (of the order 300), a conclusion was drawn in [16] that in this comparatively young system (the age is of the order of 2.10^7 [22]) all or nearly all stars fainter than visual magnitude 13.3 are flare stars. This first and naturally quite rough statistical estimation of the total number of flare stars in the Pleiades was based on data of 60 flare stars known by 1968. Subsequent investigations [4, 17], based on richer observational data made this conclusion more substantial and precise. For the lower limit of the total number of flare stars in the Pleiades, an estimate of the order of 700 was obtained. Evidence favoring great, yet slow changes in the activity of some flare stars in the Pleiades [4] is conducive to the conclusion that the total number of flare stars is considerably larger than the estimations made.

Owing to the intense regular observations of the Pleiades region made largely at Asiago, Byurakan and Tonanzintla, the number of known flare stars in this region of the sky has thus far exceeded 207 [4], providing splendid proof of the conclusion on the abundance of flare stars in this system.

Flare stars in profusion are also observed in the Orion association. The total number of flare stars in this system is estimated to be of the order of 1000 [17].

These results, testifying to the fact that the number of flare stars is comparable to the total number of stars of low luminosities in those systems, show unequivocally that the flare star stage is a natural stage in the life of stars.

Relation between the stages of RW Aurigae and the flare star

The existence of nonstable stars, possessing simultaneously the properties of the T Tauri-type star and those of the flare star, attests that those stages in the evolution of dwarf stars sometimes mutually overlap in time. Haro [23] pictures their gradual transition in the form of the following evolutionary sequence:

1. T Tauri-type stars in which the flares superimpose with irregular changes (examples: DF Tau, YZ Ori, BW Ori).
2. Dwarfs of the later type in which spectral characteristics of the T Tauri-type star still occur, although quite reduced, and the flares remain as most remarkable changes (examples: V 389 Ori, V 390 Ori).
3. "Pure" flare stars, in which properties of the T Tauri-type virtually disappear, at least during prolonged periods of a more or less constant minimum (examples: EY Tau, FF Tau, FH Tau, V 386 Ori, V 498 Ori).

To determine the evolutionary relation between the stage of RW Aurigae and the stage of flare star, one of the authors [24] estimated the total number of RW Aurigae-type variable stars in the Orion association, which have experienced flares, based on a sample of this class of stars in this system. It was shown that only one-fourth of all the RW Aurigae-type stars in the Orion system experience flares with an amplitude exceeding $0^m.5$. In view of the fact that the RW Aurigae stage is much younger than the flare star stage the conclusion was drawn to the effect that flare activity starts only shortly before the end of RW activity. However, a review of this concept might be necessitated in the light of the possible recurrence of flare activity of the RW Aurigae-type stars [4].

Spectra of flare stars

The spectra of all the flare stars in the intervals between the flares belong to the later K and M classes. The earliest spectral class of flare stars in stellar systems correlates with their age. In the quite young systems of Orion and NGC 2264, the earliest class is K 0; in the older system of the Pleiades, it is K 3 and attains as much as M 0 – M 6 in the rather old systems of Hyades, Praesepe and Coma as well as among flare stars in the vicinity of the Sun [25, 26].

In general the spectra of flare stars outside the flares differ from the spectra of normal dwarf stars by the presence of emission lines of different intensities. In comparatively low flare activity only the lines Ca II are observed in the emission, whereas in higher activity the Balmer lines of the hydrogen series are observed as well.

However, the spectra undergo radical changes during a flare. During a flare, the spectral characteristics of flare stars almost completely coincide with the specific features observed in T Tauri-type stars: apart from a bright intense line spectrum a strong continuous emission is present, especially in the ultraviolet.

Thus the ability of a star to produce flare correlates with the presence in its spectrum of emission lines in the time of minimum brightness, testifying to their chromosphere activity. It was also made clear that the intensity of emission lines is reduced as the corresponding system advances in age (Orion-Pleiades-Hyades) [26].

H-R diagram of flare stars

On the Hertzsprung-Russell diagram flare stars fall in a region that coincides in some measure with that taken up by T Tauri stars.

On the diagram (V, B–V), compiled for the Orion association, flare stars are found with approximately 13.5 visual magnitude [18] and fainter. All the bright flare stars are located above the main sequence when deflections from the latter do not exceed 1^m in the blue region of colors and attain 4^m in the red region. Flare stars fainter than 16^m are uniformly distributed on either side of the main sequence and occupy an area of up to several magnitudes on either side [18].

The basic difference between the diagrams of flare stars in the Pleiades and in Orion lies in the fact that in the former deflections from the main sequence are considerably less, while the brighter flare stars of the cluster are absolutely fainter and have a later type spectrum than in Orion. This difference is more pronounced in the case of the clusters Hyades, Praesepe and Coma [18].

The presence of an appreciable number of stars *below* the main sequence forms a characteristic and very important feature of the diagram (V, V–B) of flare stars, particularly of the Orion association. In this sense it recalls the diagram drawn up by P. P. Parenago [27] for stars of the cluster of the Orion nebula in the region of low luminosites.

Although considerable errors in determining the magnitudes and colors of faint stars might have had their possible influence on the diagram, Haro believes [18] that both in Orion and in NGC 2264 faint flare stars exist that certainly are located much below the main sequence.

The existence of stars located on the Hertzsprung-Russell diagram in the region below the main sequence is confirmed by spectroscopic observations of the Pleiades and Hyades stars, made by G. Herbig [28], as well as by multicolor photographic observations of flare stars in Orion, made by A. Andrews [29]. It was made clear, for instance, that in this system out of 19 flare stars possessing large ultraviolet and blue excesses, 14 lie below the main sequence [29] on the diagram (V, B–V). This question needs further elaboration in view of its significance for stellar evolution.

Amplitudes of flares

The investigations of certain flare stars close to us (UV Ceti and others) have made it possible to determine the approximate law of the distribution of the amplitudes of flares. It turned out that flare frequency increases as the amplitude decreases. However, this growth takes place in such a slow way that the mean total energy, radiated by all the flares in photographic rays, appears to be considerably less than the normal radiation of the star over the same period [30, 31].

The largest flares have been observed in photographic rays in some stars in the Pleiades. Of the photographic amplitudes observed in the Pleiades the largest equals about 7^m [32]. In some cases amplitudes equal to 7^m and in one case in Orion an amplitude exceeding 8^m [33] have been observed in U-rays. In fact, photographic observations of flares in associations and clusters are of low resolving power in time. Thus in Tonanzintla, where observations in U-rays are effected, 15-minute exposures are practiced. Meanwhile, the duration of the maximum itself should be much less. Therefore, the observed values of the amplitudes need correction by $2-3^m$. In other words, the true maximum amplitudes of flares in U-rays possibly attain 10^m.

The available photographic observations of flares cover only flares with amplitudes $\geq 0^m.5$. Meanwhile, as noted above, flares of great amplitude (power) are far rarer than flares of small amplitudes [31, 34]. On the other hand, observations of flares are relatively complete only for bright stars. In the case of flare stars fainter than the limiting magnitude of the telescope in the minimum, we are deprived of considerably larger amplitudes as well.

Therefore, the observed distribution of amplitudes of flares in stellar aggregates is quite distorted. In particular, a sharp increase of the mean amplitude in the transition to faint stars, discovered for the Pleiades [17], is, no doubt, due appreciably to the selection of observations.

Despite the paucity of observational material, the data available suggest a direct correlation between the amplitude and the duration of flares (minimum-maximum-minimum): with an increase of the amplitude of the flare its average duration increases [35, 36].

It should be added that in most cases the amplitude of a flare grows toward the ultraviolet.

Frequencies of flares

The statistics of flares, observed in the brightest stars of the UV Ceti type, show [31] that the sequence of flares in an individual star is adequately expressed by Poisson's law. The average frequency of flares is, in general, different for various flare stars.

For instance, according to a statistical study [31], based on data of photoelectric observations of the UV Ceti and YZ Canis Minoris, the average frequencies of flares with amplitudes $\geq 0^m.15$ in those stars are equal to 0.7134 hours^{-1} and 0.2274 hours^{-1}, respectively. The average frequencies of flares in flare stars within a stellar aggregate also differ markedly. Thus, the flares observed with an amplitude $\geq 0.^m5$ in the Pleiades are expressed adequately by the superposition of Poisson distributions with two different frequencies differing by more than one order of magnitude [4, 17]. However, observational data evidences changes in the frequencies of flares in flare stars in the course of time. According to [17], the frequency of flares increases on the eve of the cessation of their flare activity. Haro suggests [18, 25] that the frequency of the flares is, on the average, greater in older systems.

Energies of flares

In accordance with Kunkel's estimation [30], the activity of flare stars in the vicinity of the Sun (UV Ceti-type stars) has an upper limit: the averaged in time energy liberated during flares does not go beyond 1% of the energy radiated by the photosphere of the star. If this conclusion, corroborated in the investigation of V. S. Oskanian and V. Yu. Terebizh [31], is valid, one should assume that the flares of flare stars, despite their high intensity, are insignificant in the energy balance of the star because of their short duration and low frequency.

Calculation indicates that continuous and irregular variations in the brightness of the T Tauri stars are considerably more efficient, as far as energy is concerned. The energies, accounting for the variations in the brightness of the T Tauri stars, are comparable with the energy of their total radiation.

It should be noted that in both cases the matter concerns those energies that have manifested themselves in the form of optical radiation. In the meantime, we already know from solar flares that a considerable part of the energy is spent on the formation of particles of high energy (protons of cosmic rays) emitted into surrounding space. If we adopt the interpretation of flares as explosions, going on above the photospheric layers, then we have to admit (see further) that at least in some cases the optical energy of flares forms only a small part of the total energy of the corresponding explosion. The total energy released during flares can apparently exceed the energy contained in optical flares, at least by one or two orders.

Proper colors of flares

The color of the excessive radiation due to flares can be derived from the observed colors of the total radiation of the flare star in the maximum and minimum brightness [37]. Observations show [38] that most of the flare stars in the minimum have the normal colors of B–V, corresponding to their absorption spectra.

The proper colors of flares, B–V and U–B, determined for a number of flare stars in the vicinity of the Sun and also for the member of the Pleiades H II 1306 [34, 37], differ only slightly from each other and are quite at variance with the colors of blackbody radiation. This fact can apparently be regarded as a proof of the general nonthermal nature of the excessive radiation, accounting for the flare, depending only loosely on the physical parameters of the star, and especially on its effective temperature.

It should be pointed out that the proper colors of flares change considerably as the flare intensifies or dies away, remaining nearly all the time on the two-color diagram (U–B, B–V) above the curve representing the colors of blackbody radiation for various temperatures [34].

Physical nature of flares

The problem of the nature of flares is of particular interest in matters of stellar evolution. It is closely related to the problem of the sources of the energy of flares.

According to a hypothesis advanced and substantiated by one of the authors of the present paper [10, 11], the continuous emission usually present in spectra of the T Tauri-type stars and appearing in the spectra of flare stars only during the flare is of nonthermal nature. We ascribe this fact to the comparatively young age of the above stars and explain it in terms of the ejection into the outer layers of the star of certain portions of pre-stellar matter and the release of energy they have brought into those layers. The process of flare, that can often occur high above the photospheric layers, results from the decay of this matter, and elementary processes must take place similar to those that are generally observed in nuclear decay.

The first part of this hypothesis (the Byurakan hypothesis) is supported by later investigations. Thus, for instance, in the works of M. A. Arakelian [39] and one of the present authors [37, 40], certain evidence, based on the analysis of observational data concerning the flares, was obtained in favor of the nonthermal and nonsynchrotron nature of continuous emission, due to the flares. This concept is apparently supported by the fact that optical flares of the UV Ceti-type stars, at least the intense ones, are accompanied by radio flares, also of a nonthermal nature [41].

Numerous attempts to explain the continuous emission by the known mechanisms of radiation failed [40, 42]. Scrutinizing and rejecting most of the existing interpretations of flares, R. E. Gershberg [42], arrives at the conclusion that the totality of optical observations is in line with the nebular hypothesis.

The nebular hypothesis, elaborated by Gershberg [43, 44] and also by Kunkel [34], assumes that the radiation of flare stars during the flares is a superposition of the radiation of the star and of a hot, ionized, and rapidly emitting gas mass ejected by the star. However, the nebular hypothesis also encounters serious difficulties [37] when it comes to an interpretation of the proper colors of flares, i.e., continuous emission. To overcome those difficulties, Kunkel [34] has considered another component of radiation, presenting the total radiation of the star during the flare as the combined radiation of cold star, hot gas envelope and heated hot spot on the surface of the star. The rapid rate of flares is thus far an insurmountable obstacle for the nebular model. The latest observations of S. Cristaldi and M. Rodono [8], made with high resolving power in time, show that the duration of a flare (minimum–maximum–minimum) can be less than 15 sec.

As to the second part of the Byurakan hypothesis [10, 11], it should be noted that in all the interpretations of continuous emission by known mechanisms of radiation, the question of the origin of energy remains open. The presence or appearance of unknown sources of energy in the star, heavily concentrated and localized not far from its surface, is always presumed. This is virtually the initial assumption of the Byurakan hypothesis (see, for instance, [40]).

It should be added that estimations of the energies accounting for the flare and for irregular variations of the T Tauri-type stars, presumably testify to the fact that the activity, produced by the nonstability of a young star, dies away with age. Presumably the complete extinction of flare activity begins when the supplies of energy are consumed.

"Fast" and "slow" flares

As noted above, the duration of a flare (minimum-maximum-minimum) depends, on the whole, on the amplitude. However, following Haro [19, 23], two essentially different types of flares can be distinguished: "fast" and "slow," which are, respectively, shorter and longer than 30 minutes. Because of the low resolving power in time of photographic observations, the threshold of 30 minutes is determined by the conditions of the observations of Haro and his associates [19, 23] who used 15-minute exposures in the U-rays.

It is highly important that in such a division into "fast" and "slow" flares the type of flare is independent of the amplitude: "fast" flares with great amplitudes and "slow" ones with small amplitudes have been observed. For instance, in the flare star FSO 7 (the flare star of Orion No. 7) a flare was recorded with an amplitude $7^m.7$ in the ultraviolet for which the maximum was attained in 23 minutes [33].

Most flares are "fast." "Slow" flares have so far been observed only in the following seven flare stars of Orion:

No.	V	B–V	References
66	$15^m.0$	$1^m.35$	[23, 29]
92	$15^m.86$	$1^m.31$	[23, 29]
149	$16^m.61$	$1^m.14$	[23, 29]
153	$15^m.56$	$1^m.32$	[23, 29]
177	$16^m.7$	$1^m.5 - 1^m.8$	[33]
229	17.6 U	–	[5]
239	19.5 U	–	[5]

In accordance with Haro's observations [19, 23], the "fast" and "slow" flares differ sharply in the nature of changes observed in the spectra. The "fast" flares are characterized by sharp changes in the radiation in U- and B-rays and strong emission lines, while in V-rays, particularly in the red region, the changes are either small or are completely lacking. In the case of "slow" flares, an intensification of the continuum in the red region of the spectrum is noticed, accompanied by low intensity of the emission lines, notably H_a.

An interpretation of the difference of "slow" and "fast" flares and the phenomenon of fuors

The differences noted in the "slow" and "fast" flares can be interpreted within the Byurakan hypothesis [10, 11] if we assume that the phenomena producing the flare take place in various layers of the star. When the excitation energy is released high above the photospheric layers, in the chromosphere or in the corona, a sudden and very rapid increase of the nonthermal shortwave (ultraviolet and blue) continuous emission occurs that causes the flare of brightness of the star. At the same time, an intense emission line spectrum appears. When the energy is released in deeper, subphotospheric layers, an increase of thermal radiation is observed in the visible region of the spectrum, together with relatively weak emission lines. The duration of the flare from the minimum to the maximum should in the latter case be much longer.

Such an interpretation of the difference between "fast" and "slow" flares suggests that the "slow" flare stars can at times experience "fast" flares too. In this connection it should be noted that out of seven "slow" flare stars in Orion, for three (FSO 66, 149, 153) "fast" flares have already been observed, while the star FSO 177 shows irregular variations of brightness outside of flares [33].

On the Hertzsprung-Russell diagram most stars which experience "slow" flares (four of the five with known colors) fall in the region above the main sequence, i.e., possess apparently more extensive photospheres than normal stars. It can be assumed that it is precisely for this reason that the probability of energy release under the photosphere ("slow" flares) is greater for them than for stars located below the main sequence.

It should be added that until recently "slow" flares have been observed only in the Orion association. Haro even concluded [23] that such flares do not occur at all in the Pleiades stars. However, E. S. Parsamian [45] has discovered a truly "slow" flare in the star FSP 103 (the flare star in the Pleiades). This fact supports the above interpretation of the differences in "fast" and "slow" flares.

The energy of the "slow" flare of the star FSO 177 was at least several dozen times larger than the maximum energy of "fast" flares in stars of the same magnitude in Orion. This fact is by now the most sound corroboration of the Byurakan interpretation. If we assume the transition of nearly all of the explosion energy into the optical region during the "slow" flare of the FSO 177, we come inevitably to the conclusion that in fast flares only about 1% of the energy is emitted into the optical part of the spectrum.

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Luminosities and masses of flare stars

Observations of flare stars in clusters and associations indicate a great variety in their luminosities. According to Haro [18], in the Orion association the visual absolute magnitude of flare stars varies in the interval of values from $+4^m$ to $+13^m$, i.e., their luminosities can differ by four orders. The brightest flare stars in the Pleiades have an absolute magnitude of $\sim +6^m$ [16].

Thus, the flare stars in associations and clusters have in most cases luminosities that considerably surpass the luminosities of flare stars in the vicinity of the Sun.

This and the spectral classes of the UV Ceti-type stars presumably attest that the latter are rather old objects: in this group the flare activity in stars of higher luminosity has long been extinct.

The masses were determined for only a small number of the closest flare stars of the UV Ceti type. They are on the average of the order of 0.1 of the solar mass. For most flare stars the masses can be estimated roughly by means of mass-luminosity ratio, the application of which is not sufficient in this case. In view of the fact, however, that the luminosities of flare stars in stellar aggregates exceed to a great extent the luminosities of the UV Ceti-type stars, we maintain that their masses must be larger, on the average, by half an order; the great dispersion of luminosities indicates the great variety of masses in those stars.

Unusual distribution of flare stars in the Pleiades

A study of the distribution of flare stars in parent systems is of significance in the problem of stellar evolution. Such a study, carried out for Pleiades by M. A. Mnatsakanian and one of the authors of [46], has revealed that flare stars are nearly completely lacking in the central region of the system. The radius of this cavity is equal to 1.4 parsec. The partial density of flare stars reaches maximum at a distance of 1.5 parsec from the center of the system; then it diminishes more rapidly than $\sim r^{-2}$. These results have been obtained under the assumption that all known flare stars in this region belong to the Pleiades system.

The existence of a cavity in the distribution of flare stars in the central region of the Pleiades can hardly be explained by the influence of the absorbing matter.

A plausible explanation is that stars originating in the central part of the cluster move from the center of the system in the course of their aging.

Conclusion

Flare stars form one of the initial stages in the evolution of dwarf stars. In this connection a detailed and profound study of flare stars is of paramount importance for the problem of stellar evolution.

The conclusion of Haro [18] can be quoted to the effect that there is a considerable number of young stars located on the Hertzsprung-Russsell diagram *below* the main sequence.

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Editor's Addendum: Frequencies of Flares in a Stellar Aggregate

This brief comment concerns the problem of statistical determination of the number of "potential" flare stars in a stellar aggregate which showed no flashes during the observation period.

The mathematical model proposed by V. A. Ambartsumian in [1] makes it possible to draw conclusions about this number. The purpose of this note is to point out the remarkable robustness of the statistical estimate proposed by Ambartsumian, which was not mentioned in [1]. Axiomatically, the model is as follows:

a) Within the aggregate of stars there is an unknown number N of flare stars.

b) To each flare star a value of parameter $\lambda > 0$ is assigned. The values of λ for different flare stars are independent, identically distributed random variables. Their common probability density function $f(\lambda)$ is unknown.

c) Conditional upon its value of λ, each flare star produces flashes at moments of stationary Poisson process of rate $\lambda > 0$.

Recall that in a stationary Poisson process of flashes of rate λ the number of flashes in non-overlapping time intervals is independent and that the probability of k flashes in a time interval of length τ is

$$\frac{(\lambda\tau)^k}{k!}\, e^{-\lambda\tau}, \quad k = 0, 1, 2, \dots .$$

Let $p_k(\tau)$ be the probability that for a typical flare star the number of flashes observed during the time interval will be k. Clearly

$$p_k(\tau) = \int \frac{(\lambda\tau)^k}{k!}\, e^{-\lambda\tau}\, f(\lambda)\, d\lambda.$$

The product

$$N_k(\tau) = N \cdot p_k(\tau)$$

equals the mean number of flare stars which produce exactly k flashes during the observation period. It is natural to take the random quantity

$$\begin{aligned} n_k = {} & \text{the number of stars which produced } k \text{ flashes} \\ & \text{during the observation period} \quad (0, \tau). \end{aligned}$$

as an estimate of $N \cdot p_k(\tau)$.

This we express by writing

$$n_k \approx N \cdot p_k(\tau). \tag{1}$$

This estimate becomes reasonable for larger values of τ. The problem is now formulated: *find an estimate for $N_0(\tau)$ in terms of the numbers n_k, $k > 0$.*

We will consider the values

$$n_1 = 123, \quad n_2 = 16, \quad n_3 = 2, \tag{2}$$

which have been observed in Pleiades, see [1].

An example of an estimate for $N_0(\tau)$ can be obtained in the following way:

We write an identity

$$N_0(\tau) = N \cdot p_0(\tau) = \frac{(N \cdot p_1(\tau))^2}{N \cdot p_2(\tau)}\, c(\tau) = \frac{(N_1(\tau))^2}{N_2(\tau)}\, c(\tau),$$

where

$$c(\tau) = \frac{p_0(\tau)\, p_2(\tau)}{(p_1(\tau))^2}.$$

Replacing $N_1(\tau)$ and $N_2(\tau)$ by the observed quantities n_1 and n_2, we obtain

$$N_0(\tau) \approx \frac{(n_1)^2}{n_2}\, c(\tau). \tag{3}$$

The presence of the factor $c(\tau)$ which depends on the unknown density $f(\lambda)$ bars (3) from direct application. However, $c(\tau)$ can be calculated for special choices of $f(\lambda)$. Thus, it was noted in [1] that

$$c(\tau) \equiv \frac{1}{2}, \quad \text{if} \quad f \quad \text{is} \quad \delta\text{–type, i.e.,} \quad f(\lambda) = \delta_{\lambda_0}(\lambda), \text{ for some } \lambda_0 > 0, \tag{4}$$

for any choice of $\lambda_0 > 0$, as well as

$$c(\tau) \equiv 1 \quad \text{if} \quad f(\lambda) = \alpha\, e^{-\alpha\lambda} \tag{5}$$

for any choice of $\alpha > 0$.

The estimate of $N_0(\tau)$ proposed in [1] was based on (3) and assumption (4). Ambartsumian wrote a Schwartz inequality

$$\left(\int \lambda e^{-\lambda\tau} f(\lambda)\, d\lambda\right)^2 \leq \int \lambda^2 e^{-\lambda\tau} f(\lambda)\, d\lambda \int e^{-\lambda\tau} f(\lambda)\, d\lambda,$$

from which upon multiplication by $N^2\tau^2$ he obtained

$$N_0(\tau) \geq \frac{(N_1(\tau))^2}{2\, N_2(\tau)}, \tag{6}$$

i.e., $(n_1)^2/(2n_2)^{-1}$ becomes an estimate of the lower bound for the quantity in question.

It is natural to try to extend these ideas by considering a family of *double ratio estimates*

$$N_0(\tau) \approx \frac{n_k(\tau)\, n_l(\tau)}{n_m(\tau)}\, c_{k,l,m}(\tau), \tag{7}$$

where k, l, m are nonnegative integers and

$$c_{k,l,m} = \frac{p_0(\tau)\, p_m(\tau)}{p_k(\tau)\, p_l(\tau)}. \tag{8}$$

In all cases the functions $c(\tau)$ depend on the unknown density $f(\lambda)$.

Our point is that even if we assume that $c(\tau)$ is known explicitly (as in cases (4) and (5)), still all these estimates are reasonable only for values of τ large enough ($\tau \approx \infty$), so that the law of large numbers becomes valid.

Then, since $\tau \approx \infty$ is a necessary condition, it is natural to replace $c(\tau)$ in these formulae by the corresponding *asymptotic decompositions*, as $\tau \to \infty$. These decompositions may depend solely on a few parameters determining with chosen precision the behavior of the corresponding $c(\tau)$ at $\tau = 0$.

We hope to obtain in this way a simpler problem involving several unknown parameters instead of an unknown function $c(\tau)$.

Assume that at the point $\lambda = 0$ our f has a Taylor approximation

$$f(\lambda) = f(0) + f'(0)\, \lambda + O(\lambda^2) \tag{9}$$

with $f(0) > 0$.

Then by an easy analysis we find the first three terms of the asymptotic expansion of $p_s(\tau)$:

$$p_s(\tau) \approx \frac{1}{\tau} f(0) + \frac{s+1}{\tau^2} f'(0) + O\left(\frac{1}{\tau^2}\right).$$

Substituting these terms into (8) yields, after simplification,

$$c(\tau) \approx \frac{\left[f(0)\tau^2 + f'(0)\tau\right]\left[f(0)\tau^2 + (m+1)\tau f'(0)\right]}{\left[f(0)\tau^2 + (k+1)\tau f'(0)\right]\left[f(0)\tau^2 + (l+1)\tau f'(0)\right]},$$

or asymptotically

$$c(\tau) = 1 + (m-k-l)\,b + O\left(\frac{1}{\tau^2}\right), \qquad (10)$$

where

$$b = \frac{f'(0)}{\tau f(0)}.$$

The most attractive are the *robust* cases where

$$k + l = m, \qquad (11)$$

i.e., where the above reduces to

$$c(\tau) = 1 + O\left(\frac{1}{\tau^2}\right).$$

The Ambartsumian case $k = 1$, $l = 1$, $m = 2$ is robust. Taking $c(\tau) \equiv 1$, we obtain from (7) and (2)

$$N_0(\tau) \approx \frac{(n_1)^2}{n_2} \approx 946. \qquad (12)$$

In another robust case $k = 1$, $l = 2$, $m = 3$, taking $c(\tau) \equiv 1$, we obtain from (7) and (2)

$$N_0(\tau) \approx \frac{n_1 n_2}{n_3} = 984. \qquad (13)$$

The two numbers agree with 4% *precision.*

Although the estimate (13) becomes unstable for smaller values of n_3, this result confirms (12) and, what is more important, the special role of the ratio $(n_1)^2/n_2$ considered in [1].

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